



Here's our graph of $y = 3x + 1$

We're told that $a = 2$ and $b = 6$, as shown

We're asked to compute $\sum_{k=1}^4 f(x_k^*) \Delta x_k$ which

you know to represent the area of the 4 rectangles (or at least they will be 4 **rectangles** when we determine the height of each rectangle).

Part (a) asks us to use the value of the function at the left endpoint of each sub-interval.

I'll do even more, and try to explain how I translate

$$\begin{aligned} \sum_{k=1}^4 f(x_k^*) \Delta x_k &= \\ f(x_1^*) \Delta x_1 + f(x_2^*) \Delta x_2 + f(x_3^*) \Delta x_3 + f(x_4^*) \Delta x_4 &= f(2) \cdot (1) + f(3) \cdot (1) + f(4) \cdot (1) + f(5) \cdot (1) \\ &= [3(1 + \text{blue}) + 1](1) + [3(1 + \text{blue}) + 1](1) + [3(1 + \text{blue}) + 1](1) + [3(1 + \text{blue}) + 1](1) \end{aligned}$$

We could add that all together and get our left endpoint sum, but if we ever want to evaluate a definite integral – according to its definition as a limit of Riemann Sums – then we'll need to know how to write the sigma notation version

So notice that the only thing that changes from term to term in that last line is the blue part (I could have varied the expression in those inner parentheses from 2 to 6, but the way I wrote it will have k varying from 1 to 4).

So translating to sigma notation

$$\sum_{k=1}^4 3(1 + k) + 1 = \sum_{k=1}^4 3k + 4. \text{ Oh ... summing this up yields a left endpoint sum of 46.}$$

Part (b) uses the midpoint instead of the left endpoint ($5/2, 7/2, 9/2, 11/2$) and (c) uses the right endpoint.